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General Basic 0-1 Integer Programming Model for Timetabling Problems

Nur Aidya Hanum Aizam* and Thiveamalar Sithamparam

School of Informatics and Applied Mathematics,
Universiti Malaysia Terengganu, 21030 Kuala Terengganu, Malaysia

E-mail: aidya@umt.edu.my

*Corresponding author

ABSTRACT

This paper focuses on developing a general basic 0-1 integer programming model for various types of timetabling problems. Research shows that numerous models were successfully formulated to solve the timetabling problems in various fields such as job scheduling and examination timetabling. However, the models were incorporated only to the specific problems in specific field. The world of scheduling is still deficient of a general basic integer programming model which can be employed to solve all types of timetabling problems. Therefore, it would be beneficial to have a model which could be applied to problems across different field. This paper analyzes the common basic constraints in different timetabling problems prior generating a general basic model. Randomly generated data which represent the real world data will be used to test and verify the model. Models are solved using AIMMS mathematical software with CPLEX 12.6 as the solver. Results show that the model generated is well performed to the selected timetabling problems in terms of CPU time with solutions to optimal.

Keywords: Timetabling Problems, AIMMS Mathematical Software, CPLEX 12.6.

1. Introduction

The necessity of planning and scheduling is becoming extremely important in global business arenas. The organizational strategy is no longer on the market sales but how they optimize their resource and emerge as cost-efficient organization (Elmuti, 2003). Hence, scheduling is found to be an ideal method to achieve their goal. Constructing a plan to execute a number of activities over a period of time with limited resources and various constraints is simply known as scheduling (M Akif & Cihan, 2008). Scheduling problems usually arises when one want to obtain solution for the most favourable schedule under various objectives (Hochbaum, 1999). These objectives can be from optimizing profits to preferences of assigning activities to specific time.

Today almost every field need an optimal schedule in order to work efficiently, meets the customers' demand and gain its maximum profit. Hence, the use of the model constructed was restricted to that particular problem. It is obviously a waste of resources, time and energy to repeat the whole procedure of solving the scheduling problem each time in different institution. We have previously constructed a general model specifically for university course timetabling problem, and were applied successfully on sets of literature data (Aizam and Caccetta, 2014). It is essential to have a model which could solve different type of scheduling problems. The main goal of this research is to gather similar requirements and policies from various scheduling problems in different field to form a new general basic model. This is an initial step to produce a model which is applicable for all types of scheduling problems and consequently benefit timetabling construction by avoiding application in a specific field only. For this research, we focus on three different problems; nurse, university course and examination timetabling problem.

In this paper, a review of previous studies from three different timetabling problems is discussed in second section. Definition of the problem is detailed in the third section, where all notation and terminology is given. The following section is devoted to the MILP model. In this section, similar requirements and policies from three different problems are included. Example of experimental data tested on the model is given in the next section. For the subsequent section, the computational results obtained after implementing randomly generated data based on real applications towards the model is described. Results are then discussed. Finally, we conclude the paper with some remarks.

2. Literature Review

Numerous methods were tested to obtain the best schedule without violating the variety of important constraints. These methods can be categorized into two types, the exact optimization method and the heuristic optimization method. The exact method gives certainty of an optimal solution whereas heuristic method does not promise an optimal solution (Rothlauf, 2011). Exact methods include naive approaches, complete (enumerative) search strategies and successive approximation (relaxation) methods. Meanwhile, examples of heuristic methods are evolution strategies, simulated annealing, Tabu search, and approximation convex global underestimation (Pinter, 2014). According to Genova and Guliashki, 2011, the evolution of exact optimization method was very successful for the past 50 years. However, exact optimization method turns out to be very difficult and takes a longer time to obtain the optimal solution when it comes to large size optimization problem. This results in researchers moving towards other approaches. On the other hand, this problem could be overcome by choosing the right value for the cost coefficient. For this it can significantly reduce the computational time which leads to a faster optimal solution. Furthermore, the existence of mathematical programming software with IP solvers that are available these days have resulted in immediate implementation, even for large department. The employment of this software can be found in MirHassani, 2006, Aizam and Liong, 2013 and Aizam and Caccetta, 2014.

2.1 Constraints

Constraints that are related to the timetabling problems are classified as hard and soft constraints. Hard constraints are the ones that have to be met and must not be violated in any situation. Conversely, the soft constraints are the desirable type of constraints which can be treated as non-essential but stimulates circumstances that are optional. Some constraints can be unique to its own specific problem. Constraints that are involved in Nurse Scheduling Problem can be found in Choy and Cheong, 2012; University Course Timetabling Problem as in Wormald and Guimond, 2012 and Examination Timetabling Problem in Wang et al., 2010. We selected the similar requirements from all these research to be compiled and formulated into one general basic model.

3. Problem Definition

Numerous researches have been done to solve scheduling problems. However, most of the researchers restrict their research to a particular field. For instance, a particular research to solve Nurse Scheduling Problem (NSP) focuses only on the constraints that appears in NSP and hence provide a model which is only applicable to solve that particular problem. The solving for different scheduling problem using the same model apparently becomes impossible as the constraints varies.

Eventually, the whole procedure of solving scheduling problem has to be repeated in order to optimize a different scheduling problem. So as to save the cost of such repetition, a single model could be developed to solve multiple scheduling problems. The model, better be known as general basic model contains the common basic constraints of a few scheduling problems. In here, the basic constraints will be analyzed from nurse scheduling, university course scheduling and examination scheduling. Thus, to make it easy and cost-efficient, without focusing on a specific scheduling problem a general basic model which is applicable to all type of scheduling problem will be constructed. The construction of the model is based on four main constraints with the objective of minimising the preference cost of assigning course/nurse/exam to timeslots. Finally, the model constructed are tested with the data generated using AIMMS 3.13. The main common constraints found in this study are listed below:

Hard Constraints

1. All nurses/courses/exams must be in the timetable (constraint of completeness).
2. No student takes two exams/courses at one timeslot and no nurse is allowed to work more than a single shift per day (constraint of conflict).
3. No student takes 3 consecutive exams/courses and given any 3 consecutive work days within 7 days period, there must be at least 1 rest day for nurses.
4. The minimum number of nurses/exams/course must be fulfilled for each shift/timeslot.

Soft Constraints

1. The nurses'/lectures'/invigilators' preferences must be fulfilled.

The notation, formulation and objective function are given in the next section.

4. Problem Formulation/Modelling

We used binary integer programming to build the general basic model. The following notations are needed to describe the model.

4.1 Notation

Sets

C	Total number of nurses/courses/exams to be scheduled
T	Total number of timeslots/shifts
D	Total number of days
S	Total number of students

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c	Course/ nurse/exams
d	days
s	students
t	timeslot/shift

Parameters

n_{con}	number of consecutive timeslots preferred
n_d	number of timeslots in day d
$P_{c,t}$	preference of course/nurse/exams are assigned to timeslot/shift t
C_t	number of nurses/courses/exams required on timeslot/shift t

Decision variable

$$X_{c,t} \begin{cases} 1 \; if \; course \; or \; nurse \; or \; exams, c \; are \; assigned \; to \; timeslots \; or \; shifts, t \\ 0 \; otherwise \end{cases}$$

4.2 Objective Function

Minimize

$$Z = \sum_{c}^{C} \sum_{t}^{T} P_{c,t} \, X_{c,t}$$

Here, the objective function is to minimize the timeslot preference, $P_{c,t}$ of allocating each activities. Using the above notation, the objective function and the constraints take the following form:

Minimize

$$Z = \sum_{c}^{C} \sum_{t}^{T} P_{c,t} \, X_{c,t}$$

Subject to

$$\sum_t^T X_{c,t} = 1 \qquad \forall c \tag{1}$$

$$\sum_c^C X_{c,t} \leq 1 \qquad \forall t \tag{2}$$

$$\sum_c (X_{c,t} + X_{c,t+1} + \cdots + X_{c,t+ncon}) \leq ncon \ \forall\, t \in \{t_1, t_2, \dots, t_{nd-ncon}\} \tag{3}$$

$$\sum_c^C X_{c,t} \geq C_t \qquad \forall t \tag{4}$$

$$X_{c,t} \in \{0,1\} \qquad \forall c \ \ \forall t \tag{5}$$

Constraint (1) guarantees the completeness of the schedule where all courses, nurses or exams must be assigned in a timeslot. The conflict type of constraint is expressed in Constraint (2). It ensures that no student is assigned to multiple exams or courses at a time whereas for nurse, any nurse is not allowed to work more than a single shift a day. Constraint (3) ensures that no student is assigned to 3 consecutive numbers of exams or assigned to 3 consecutive classes a day. Constraint (4) is a requirement of having at least a minimum number of activities in that particular timeslot. Finally, Constraint (5) shows that the decision variables are binary.

The value of coefficient $P_{c,t}$ expresses the desirability of the assignment designed by variable $X_{c,t}$. If all the coefficients $P_{c,t}$ are 0, then a feasible solution - if one exists - is produced. In some tests the $P_{c,t}$ coefficients were designed to induce a compact schedule by guiding, through high $P_{c,t}$ values, the assignment of compulsory and optional courses belonging to a stream to specific neighboring time groups. This to some extent offsets the inability of the IP model to take care of the desirable relative position of the subjects. In this paper, the preference parameter used in the model contains the preference concerning assignment of activities (class meetings, exams and nurse) to slots. Integer values between scale 1 (least preferable) and 5 (most preferable) is used to determine these assignments. This model aim to minimise the preferences cost of each nurse, university course or examination to the timeslots. The scheduling system of this problem will consider the nurses', lecturers', and invigilators' preferences, one way to bring out the better quality service from them.

4.3 Experimental Data

We test and verify the model with data that are randomly generated representing the world data. Each type of scheduling are tested with two sizes of data, a small and medium size data. Below (Table 1-3) are data used for the respective problems:

TABLE 1: Data of Nurse Scheduling

Sets	Notation	Data (small)	Data (medium)
Total number of nurses, N	C	24	54
Total number of nurses to be assigned (each nurse works 6 times a week)	$\{1..6\} \in cn1$ $\{7..12\} \in cn2$ $\{13..18\} \in cn3$ $\{19..24\} \in cn4$	6	9
Total number of shifts, T	T	21	21
Total number of days, D	D	7	7
Total number of shifts, t per day	$\{1..3\} \in d1$ $\{4..6\} \in d2$ $\{7..9\} \in d3$ $\{10..12\} \in d4$ $\{13..15\} \in d5$	3	3
Maximum number of working day	n_d	6	6
Number of consecutive days preferred	n_{con}	3	3
Preference of nurse at timeslot/shift t (matrix)	$P_{c,t}$	Range : [1,5] 1 – most not preferred 5 – most preferred	Range : [1,5] 1 – most not preferred 5 –most preferred
Minimum number nurse for every timeslots	C_t	1	2

TABLE 2: Data of University Course Timetabling

Sets	Notation	Data (small)	Data (medium)
Total number of courses, N	C	24	44
Total number of students group taking course grp, S	cln n : {1..6}	6	11
Total number of courses taken by each student group	$\{1..4\} \in cl1$ $\{5..8\} \in cl2$ $\{9..12\} \in cl3$ $\{13..16\} \in cl4$ $\{17..20\} \in cl5$ $\{21..24\} \in cl6$	4	4
Total number of timeslots, T	T	6	10
Maximum number of timeslots per day	n_d	4	4
Number of consecutive timeslots preferred	n_{con}	2	2
Preference of courses at timeslot/shift t (matrix)	$P_{c,t}$	Range : [1,5] 1 – most not preferred 5 – most preferred	Range : [1,5] 1 – most not preferred 5 – most preferred
Minimum number exams for every timeslots	C_t	3	4

TABLE 3: Data for Examination Timetabling

Sets	Notation	Data (small)	Data (medium)
Total number of examination, N	C	24	40
Total number of students group sitting for exam, S	csn n : {1..6}	6	8
Total number of exams taken by each student group	$\{1..4\} \in cs1$ $\{5..8\} \in cs2$ $\{9..12\} \in cs3$ $\{13..16\} \in cs4$ $\{17..20\} \in cs5$ $\{21..24\} \in cs6$	4	5
Total number of timeslots, T	t	8	10
Maximum number of timeslots per day	n_d	3	6
Number of consecutive timeslots preferred	n_{con}	1	1
Preference of exams at timeslot/shift t (matrix)	$P_{c,t}$	Range : [1,5] 1 – most not preferred 5 – most preferred	Range : [1,5] 1 – most not preferred 5 – most preferred
Minimum number exams for every timeslots	C_t	3	3

The preference parameter used in the model contains the preference concerning assignment of activities (class meetings, exams and nurse) to slots. Integer values between scale 1 (least preferable) and 5 (most preferable) is used to determine these assignments.

5. Result and Discussion

In this section, the results obtained using AIMMS mathematical software with CPLEX 12.6 as the solver is presented.

TABLE 4: Nurse Scheduling

Result	Small Size	Medium Size
Iteration	215	334
Solving time	0.06 sec	0.11 sec
Objective Function	72.0	162.0

TABLE 5: University Course Timetabling

Result	Small Size	Medium Size
Iteration	51	123
Solving time	0.03 sec	0.05
Objective Function	72.0	132.0

TABLE 6: Examination Timetabling

Result	Small Size	Medium Size
Iteration	15	61
Solving time	0.25 sec	0.27 sec
Objective Function	72.0	120.0

Basic model consists of features that are commonly included. Features such as completeness and conflicts among resources (nurses, lecturers, students, rooms, etc) are examples of basic criteria that one must have in their timetabling model. For purpose of illustrating the models, randomly generated data are employed. Two entities that we should be looking at from a derived solution: CPU time on getting a solution and the satisfaction level of the timetabling communities. In this case, the timetabling community satisfaction level is the nurses', lecturers' or students' preference in having activities to a slot. As for the CPU time, the faster time to obtain a solution the better it is. However, we have to consider the size of the problem to be solved. The CPU time is also influenced by the requirements needed. As for the satisfaction level, we analyzed each assignment of activities to slots whether it has satisfied the timetabling communities need. Values are assigned to the costs coefficients in the objective function.

Successful outcomes are obtained for all the experiments in terms of the CPU time whilst satisfying the basic features that are included. Optimal solution to both small and medium size data for each of the respective nurse, university course and examination timetabling were found to be below 0.30 seconds. A fast CPU time in obtaining a solution indicates very impressive achievements. Another way of assessing the timetable produced is by measuring the cost satisfaction level of having items or activities to their preferred time slots. Integer values between scale 1 (least preferable) and 5 (most preferable) were used to determine these assignments. Satisfaction level of the timetabling communities is indicated by the preference of having activities to a slot. Values are assigned to the cost coefficients in the objective function. Each assignment of the activities is analyzed. For our model, the assignments for all three problems were given 100% to its preferred slot. This is a positive indication of producing a good timetable. We attained very encouraging results where the outcome is not only conflict free, but was also attained in a short period of time. In addition, the determined schedules respect the soft constraint related to the preference of having events to timeslot and all hard constraints listed.

6. Conclusion

In summary, we have successfully presented a general basic model for three different timetabling problems; nurse, university course and examination timetabling problems. This outcome will benefit scheduler where the model may be useful to be applied on the process of timetabling construction. The general basic model constructed enforces four hard constraints. These hard constraints are analyzed and filtered from numerous references of each problem. Implementation of randomly generated data to the model for the purposes of validation was also found to be successful. None of the hard constraints were violated to get a feasible solution. Results of the assessment on the model were found to be effective. The allocations were given to the preferred slots. The preferences of nurses, lecturers and students were satisfied to a sufficient degree. This illustrates the ability of the model application on the case study, and suggests the prospect of its applicability on other problems. To sum up, by utilising general basic model in all three nurses, university course and examination timetabling problems, the best result is achieved.

At the end of this study, it is known that both tested data gives out solution in a short time but it is claimed that the exact method takes up longer time as the complexity of the problem increases. Hence in future work it is good to test the model with a bigger datasets or perhaps on real world problems. We are also interested in extending the developed model by further analyzing more common features on all other timetabling problems.

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